

$$\begin{aligned}
 & (x^2 - 1) \cdot \left(\frac{1}{x-1} - \frac{1}{x+1} - 1 \right) = \\
 & = (x+1) \cdot (x-1) \cdot \left(\frac{1 \cdot (x+1) - (1 \cdot (x-1)) - (1 \cdot (x+1) \cdot (x-1))}{(x-1) \cdot (x+1)} \right) = \\
 & = (x+1) \cdot (x-1) \cdot \frac{x+1 - x+1 - x^2 + 1}{(x-1)(x+1)} = \\
 & = \underline{\underline{-x^2 + 3}} \quad x \neq \pm 1
 \end{aligned}$$

12 3-8:59

$$\begin{aligned}
 & \left(\frac{x}{x+1} \right)^2 \cdot \left(\frac{y}{x-1} \right)^2 \cdot \left(\frac{x^2-1}{2xy} \right)^2 = \\
 & = \frac{x^2}{(x+1)(x+1)} \cdot \frac{y^2}{(x-1)(x-1)} \cdot \frac{(x^2-1)(x^2-1)}{4x^2y^2} = \\
 & = \frac{x^2}{(x+1)(x+1)} \cdot \frac{y^2}{(x-1)(x-1)} \cdot \frac{(x-1)(x+1)(x-1)(x+1)}{4x^2y^2} = \\
 & = \frac{x^2 \cdot y^2}{4x^2y^2} = \underline{\underline{\frac{1}{4}}}
 \end{aligned}$$

$x \neq 0$
 $x \neq \pm 1$
 $y \neq 0$

12 3-9:07

$$\frac{2x^2}{3y} : \frac{4x^2}{15y^2} = \frac{\cancel{2x^2}}{\cancel{3y}} \cdot \frac{\overset{5y}{\cancel{15y^2}}}{\cancel{4x^2} \overset{2}{}} =$$

$$= \frac{\underline{\underline{5y}}}{2} \quad x, y \neq 0$$

12 3-9:14

$$\frac{12a^2b^2}{14x^2y^3} : \frac{18a^2b}{21x^3y^2} = \frac{\cancel{12a^2b^2}}{\cancel{14x^2y^3}} \cdot \frac{\overset{3}{\cancel{21x^3y^2}}}{\overset{2}{\cancel{18a^2b}}} =$$

$a, b, x, y \neq 0$

$$\frac{12b}{14y} \cdot \frac{21x}{18} = \frac{\cancel{6}b}{\cancel{7}y} \cdot \frac{\cancel{7}x}{\cancel{6}} = \underline{\underline{\frac{bx}{y}}}$$

12 3-9:17

$$\begin{aligned}
 & \frac{(a+b)^2}{a^2-b^2} : \frac{a^2+b^2}{(a-b)^2} = \frac{(a+b)^2}{a^2-b^2} \cdot \frac{(a-b)^2}{a^2+b^2} = \\
 & = \frac{(a+b)\cancel{(a+b)}}{(\cancel{a+b})(\cancel{a-b})} \cdot \frac{(\cancel{a-b})(a-b)}{a^2+b^2} = \frac{a+b}{1} \cdot \frac{(a-b)}{a^2+b^2} = \\
 & = \frac{(a+b)(a-b)}{a^2+b^2} = \frac{a^2-b^2}{\underline{\underline{a^2+b^2}}}
 \end{aligned}$$

$a \neq \pm b$

$a^2 - b^2 \neq 0$
 $a^2 + b^2 \neq 0$
 $(a-b)^2 \neq 0$

12 3-9:21

$$\begin{aligned}
 & \frac{x^3 - x^2 y}{y + y^2} : \frac{y^3 - y^2 x}{xy + x} = \frac{x^3 - x^2 y}{y + y^2} \cdot \frac{xy + x}{y^3 - y^2 x} = \\
 & = \frac{x^2(x-y)}{y(1+y)} \cdot \frac{x(\cancel{y+1})}{y^2(y-x)} = \frac{-x^2(\cancel{y-x})}{y} \cdot \frac{x}{y^2(\cancel{y-x})} = \\
 & = \frac{-x^3}{\underline{\underline{y^3}}}
 \end{aligned}$$

$y \neq 0$
 $y \neq -1$
 $y \neq x$
 $x \neq 0$

12 3-9:31

$$\begin{aligned}
 & \left(r + \frac{r-r}{1+mr} \right) : \left(1 - \frac{r \cdot (m-r)}{1+mr} \right) = \\
 & \frac{r + \frac{r-r}{1+mr}}{1+mr} : \frac{1+mr - \frac{r \cdot (m-r)}{1+mr}}{1+mr} = \\
 & \frac{r + \frac{r-r}{1+mr}}{1+mr} : \frac{1+mr - \frac{r \cdot (m-r)}{1+mr}}{1+mr} = \\
 & \frac{r + \frac{r-r}{1+mr}}{1+mr} : \frac{1+mr - \frac{r \cdot (m-r)}{1+mr}}{1+mr} = \\
 & \frac{r + \frac{r-r}{1+mr}}{1+mr} = \frac{r(r^2+1)}{r^2+1} = \underline{\underline{r}}
 \end{aligned}$$

$$mr \neq -1$$

12 4-10:07

$$\begin{aligned}
 & \left(\frac{4x^2y}{6a^2b^3} \cdot \frac{8xy^2}{12a^2b^4} \right) = \frac{4x^2y}{6a^2b^3} \cdot \frac{8xy^2}{12a^2b^4} = \\
 & = x \cdot \frac{2b}{2a} = \underline{\underline{\frac{bx}{a}}}
 \end{aligned}$$

(a;b;x;y ≠ 0)

12 4-10:15

$$\frac{\frac{x^2-9}{x^2+5x+3}}{\frac{x^2-4x+3}{x^2-1}} = \frac{\cancel{(x+3)} \cdot \cancel{(x-3)} \cdot \cancel{(x+1)} \cdot \cancel{(x-1)}}{\cancel{(x-3)} \cdot \cancel{(x+1)} \cdot \cancel{(x-3)} \cdot \cancel{(x+1)}}$$

$$= \underline{\underline{1}}$$

$x \neq \pm 1$
 $x \neq \pm 3$

12 4-10:18

$$\frac{4-x^2}{\frac{x^2-x-2}{x^2+3x+2}} = 4-x^2 : \left(\frac{x^2-x-2}{x^2+3x+2} \right) =$$

$$(2-x)(2+x) : \left(\frac{(x-2) \cdot \cancel{(x+1)}}{(x+2) \cdot \cancel{(x+1)}} \right) =$$

$$(2-x)(2+x), \frac{x+2}{x-2} = \cancel{(2-x)}(2+x) \cdot - \frac{x+2}{\cancel{2-x}} =$$

$$x \neq \pm 2 \quad \underline{\underline{-(2+x)^2}}$$

$$x \neq -1$$

12 4-10:27

$$\begin{aligned}
 \frac{1 - \frac{1}{x}}{\frac{x-1}{3}} &= \left(1 - \frac{1}{x}\right) : \frac{x-1}{3} \\
 &= \left(1 - \frac{1}{x}\right) \cdot \frac{3}{x-1} \\
 &= \frac{\cancel{x-1}}{x} \cdot \frac{3}{\cancel{x-1}} = \underline{\underline{\frac{3}{x}}}
 \end{aligned}$$

$$\begin{aligned}
 x &\neq 1 \\
 x &\neq 0
 \end{aligned}$$

12 4-10:35

$$\begin{aligned}
 \frac{x - \frac{1}{x}}{x + \frac{3x+1}{x-1}} &= \left(\frac{x-1}{1} - \frac{1}{x}\right) \cdot \frac{x(x-1) + 3x+1}{x-1} \\
 \frac{x^2-1}{x} \cdot \frac{x-1}{x^2+2x+1} &= \\
 &= \frac{\cancel{x+1}(x-1)(x-1)}{x(x+1)\cancel{(x+1)}} = \\
 &= \frac{(x-1)^2}{x(x+1)} \quad x \neq \pm 1; 0
 \end{aligned}$$

12 7-8:22

$$\begin{aligned}
 \frac{x+1}{1-\frac{x}{x-1}} &= \frac{x+1}{\frac{x-1-x}{x-1}} = \frac{x+1}{\frac{-1}{x-1}} \\
 &= \frac{x+1}{1} : \frac{-1}{x-1} = \frac{x+1}{1} \cdot \frac{x-1}{-1} \\
 &= -(x^2-1) = -x^2+1 = \underline{\underline{1-x^2}} \\
 &x \neq 1
 \end{aligned}$$

12 7-8:37

$$\begin{aligned}
 \frac{1-\frac{1-1}{1+1}}{1+\frac{1(1-1)}{1+1}} &= \frac{\frac{1(1+1)-1+1}{1+1}}{\frac{1+1+1(1-1)}{1+1}} = \\
 &= \frac{(1^2+1) \cdot (1+1)}{(1+1) \cdot (1^2-1+1+1)} = \\
 &= \frac{(1^2+1)(1+1)}{(1+1)(1^2+1)} = \underline{\underline{1}} \\
 &\quad \underline{\underline{1 \neq -1}}
 \end{aligned}$$

12 8-9:04

$$\frac{x^2 + xy}{5x^2 - 5y^2} : \frac{x^2 - xy}{3x^2 - 3y^2} =$$

$$\frac{x(x+y)}{5(x^2 - y^2)} : \frac{x(x-y)}{3(x^2 - y^2)} =$$

$$x \neq \pm y$$

$$x \neq 0$$

$$\frac{\cancel{x}(x+y)}{5\cancel{(x^2 - y^2)}} : \frac{3\cancel{(x^2 - y^2)}}{\cancel{x}(x-y)} = \frac{3(x+y)}{5(x-y)}$$

12 8-9:16

$$\left(\frac{x}{x-1} - \frac{x+1}{x} \right) : \left(\frac{x}{x+1} - \frac{x-1}{x} \right) =$$

$$= \left(\frac{x^2 - (x^2 - 1)}{x(x-1)} \right) : \left(\frac{x^2 - (x^2 - 1)}{x(x+1)} \right) =$$

$$= \frac{1}{x(x-1)} \cdot \frac{x(x+1)}{1} =$$

$$= \frac{x(x+1)}{x(x-1)} = \frac{(x+1)}{(x-1)}$$

$$x \neq \pm 1$$

$$x \neq 0$$

12 8-9:21

$$\begin{aligned}
 & \left(y^2 - 2 + \frac{3}{y} \right) : \left(1 + \frac{1}{y} + \frac{1}{y^2} \right) = \\
 & = \frac{y \cdot (y^2 - 2) + 3}{y} : \frac{y^2 + y + 1}{y^2} = \\
 & = \frac{y \cdot (y^2 - 2) + 3}{y} \cdot \frac{y^2}{y^2 + y + 1} = y \neq 0 \\
 & = \frac{(y^2 - 2y + 3) \cdot y}{y^2 + y + 1} \quad y^2 + y + 1 \neq 0
 \end{aligned}$$

12 8-9:29

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12 8-9:37